THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH2040A (2024-25 Term 1)
Linear Algebra II
Course Outline

Outline

This course is a continuation of Linear Algebra I (MATH1030). It is a second course on linear algebra and will cover basic concepts of abstract vector spaces, linear transformations, eigenvalues and eigenvectors, diagonalizability, operators on inner product spaces, orthogonality and Gram-Schmidt process, adjoint, normal and self-adjoint operators, spectral theorems. *More emphasis will be put on the theoretical understanding of basic concepts in linear algebra.*

Prerequisites

Students taking this course should have taken MATH1030 *Linear Algebra I* and MATH1050 *Foundation of Modern Mathematics* (or classes at equivalent level). In particular, students are expected to have good working knowledge on the following:

- solving systems of linear equations by Gaussian eliminations
- matrix manipulations including e.g. rank, inverse, transpose and determinant
- $\mathbb{R}^n$ as a vector space, subspaces, linear span, linear (in)dependence, basis and dimension (computational aspects)
- computing eigenvalues and eigenvectors of a matrix, diagonalization
- inner product on $\mathbb{R}^n$, orthogonality, Gram-Schmidt orthogonalization process
- familiarity with complex numbers and polynomials

Class Information

- Instructor: Prof. Renjun DUAN (Office: LSB 206, Tel: 3943 7977, Email: `rjduan@math.cuhk.edu.hk`; please make an appointment by email if you have any question)
- Lectures: Monday 15:30-16:15 Mong Man Wai Bldg 710; Thursday 16:30-18:15 Mong Man Wai Bldg 710
- Tutorials: Monday 16:30-17:15 Mong Man Wai Bldg 710; Thursday 15:30-16:15 Mong Man Wai Bldg 710
- Webpage: `https://www.math.cuhk.edu.hk/course/2425/math2040a`

Teaching Assistants (Please make an appointment with TAs by email if you have any question)

- Mr. Kam Fai CHAN (Office: LSB 232, Tel: 3943 5294, Email: `akfchan@math.cuhk.edu.hk`)
- Mr. Junhao ZHANG (Office: LSB 232, Tel: 3943 5294, Email: `jhzhang@math.cuhk.edu.hk`)

Textbook

- Friedberg, Insel and Spence, *Linear algebra*, 4th edition, Pearson.

Reference

- Axler, *Linear algebra done right*, 3rd edition, Springer.
- Strang, *Linear Algebra and Its Applications*, 4th edition, Cengage Learning.

Lectures, Tutorials and Homeworks

Lectures: Students are supposed to attend **ALL** the lectures. The lectures will focus mainly on the theoretical concepts and proofs, supplemented occasionally with some illustrative examples. Below you can find a tentative schedule of this course, indicating the involved sections in the textbook that will be covered. As the lectures will only cover the most essential materials (at a rather fast pace), it will be very helpful if you have read (or at least skimmed through) the relevant sections beforehand. The full review note of the course (PDF) would be helpful for you to sketch the covered materials, and students are also strongly encouraged to take your own class notes.

Tutorials: Students are expected to attend **ALL** the tutorials. The tutorials will cover more examples and computational aspects of the materials. There will be times during the tutorials for discussions and working out some exercises together. All the materials (except otherwise stated) covered in lectures and tutorials will be covered in the midterm and final exams.

Homeworks: There will be **WEEKLY** problem assignments, usually posted on the course web on Thursdays and due on Fridays in the following week (submitted via the Blackboard online). Each problem assignment consists of two parts: the compulsory part and the optional part. **You only need to hand in your solutions of the compulsory part.** However, you are highly recommended to work out the optional part at home as well. Keep in mind that the best way to learn mathematics is to work on exercises and get the feeling by yourself. Apart from the assigned problems, the textbook has a vast collection of exercises at the back of each section. Students are highly encouraged to work out these exercises and turn to TAs if you meet any trouble. Discussions among classmates are strongly encouraged.

Assessment

- 10%: Homework (about 10 times) - submission via Blackboard
- 40%: Midterm Examination (**Time and Date: 1830-2030 Oct 25 Friday; Venue: TBA**)
- 50%: Final Examination (TBA by University)

Note: In order to pass MATH2040A, a student **MUST** write the Final Examination. Both Midterm and Final Examinations will be closed-book. You may use a non-graphical calculator, but your calculator should **NOT** contain any "programmable" equations or formulae. The use of computers/cellular phone/graphical calculators will **NOT** be permitted during both examinations.

Tentative Schedule (13 weeks in total)

- Week 1–3: Foundations of abstract vector spaces (Textbook Sec. 1.2–1.6)
  - definition of vector spaces and subspaces
  - sum and direct sum of subspaces
  - span and linear independence
  - basis and dimension
- Week 4–6: Linear maps (Textbook Sec. 2.1–2.5)
  - definition of linear maps, linear maps as a vector space
  - null space and range, rank-nullity theorem
  - matrix representation of a linear map, change of basis formula
  - invertibility and isomorphism
  - determinant and its properties (review)
- Week 7–9: Eigenvalues, eigenvectors and diagonalizability (Textbook Sec. 5.1, 5.2, 5.4)
  - eigenvalues and eigenvectors, characteristic polynomials
  - diagonalizability, algebraic and geometric multiplicity
  - invariant subspaces, Cayley-Hamilton theorem
- Week 10–13: Operators on inner product spaces (Textbook Sec. 6.1–6.4, 6.6)
  - inner products and norms, basic identity and inequalities
  - orthogonality, orthonormal basis, orthogonal complement
  - adjoint of an operator
  - normal and self adjoint operators
  - spectral theorem